

MASSACHUSETTS CURRICULUM PLACEMENT

How Many Times as Much

Grade 4 · Mathematics · Where This Fits in Your Year Place this unit after students are secure with basic multiplication and division and before, or alongside, work that asks them to compare measured quantities. The seven lessons make one distinction explicit: the same two quantities can have both an additive difference and a multiplicative relationship, and the wording determines which comparison is being asked for.

Students prove exact multiplicative comparisons with strips, describe the same pair multiplicatively and additively, reverse comparison statements, recover missing multipliers, apply them to new measurements, and test numerical claims. The culminating task requires both comparison types in both directions, with equations.

For Massachusetts, the connections below are limited to the strongest Grade 4 expectations supported by those student actions.

STRONGEST CURRICULUM CONNECTIONS - OUR SUMMARY

TEACHES · Lessons 1, 2, 3, 4, 7 4.OA.A.1 - Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.

Student evidence: Students build multiplicative comparisons, state them in words in both directions, and write equations that identify the comparison factor.

TEACHES · Lessons 2, 4, 5, 6, 7 4.OA.A.2 - Multiply or divide to solve word problems involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison.

Student evidence: Students solve contextual multiplicative-comparison situations, find and apply missing factors, test claims, and keep the multiplicative factor distinct from the additive difference.

Curriculum source: Massachusetts Curriculum Framework for Mathematics (2017) / unchanged standard per DESE revision guide <https://www.doe.mass.edu/frameworks/search/>

MATH UNIT	
How many times as much	
Grade 4 7 lessons 7 × 30 minutes 4 source books	210 minutes in all
THE QUESTION THIS UNIT FOLLOWS Times, or more?	

ABOUT THIS UNIT

Grade 4 is where comparison stops being additive. Twenty feet and four feet are sixteen apart and also five copies, and both are true of the same pair. This unit builds both relations physically, with strips laid end to end and cubes matched against cubes, and asks for both every time.

It then does the harder half, which is language. The word exactly is what turns a comparison into a measuring job. A reversal that keeps the old comparison word is the error worth catching. And a book that says twice as much never says half as much, so the second sentence has to be supplied by the reader.

THE MATHEMATICAL ARC

Each lesson does one job the lesson before it made possible.

1	Exactly twice as long, proved by laying the short one along the long one twice
2	Five fours or sixteen more: one pair, two kinds of comparison
3	Exactly twice, or more than twice: which phrase the measurement earns
4	Three times, one third, and two hundred sixty more
5	Find the multiplier the book implies, and build the length it demands
6	Use the multiplier to test a claim the book never works out
7	Twice as much and half as much, in words the book never uses

SOURCE BOOKS

Every lesson in this unit is already published under Math lessons. Nothing here asks you to teach anything you cannot open today.

Pythagoras and the Ratios 2 lessons in this unit

Beanstalk: The Measure of a Giant 3 lessons in this unit

On the Trail with Lewis and Clark 1 lesson in this unit

Follow Your Money 1 lesson in this unit

HOW TO USE THIS GUIDE

Open the source lesson before you teach.

Each day names one lesson that already exists under Math lessons, with its book, its number and its title. This guide says what that lesson is doing in the sequence and what to collect from it; the lesson itself carries the steps, the printables and the answers.

Thirty minutes a day.

Five minutes to connect to yesterday, the twenty-minute source lesson, five minutes to close on something you can see. A day that runs long should lose the close, never the core.

Teach a lesson on its own if you need to.

Every day carries a one-sentence opening for a teacher who is using that lesson outside the sequence.

Stop after a checkpoint.

The unit holds together if you stop after either checkpoint and come back to it. Stopping between a checkpoint and the next one leaves a lesson without the evidence it was built on.

WHERE THIS FITS IN YOUR YEAR

When to teach it After multiplication is secure and before or alongside work on measurement conversion. It runs for a fortnight and needs floor space.

What students need first Children can multiply and divide within one hundred, and can measure lengths with a tape.

What this unit develops Multiplicative comparison; distinguishing it from additive comparison; the exactness a comparison claims; finding a missing factor; using a comparison to test a stated claim; reversing a multiplicative relation.

What it prepares them for Ratio and scale, and any later work where a claim in prose has to be checked against a computation.

CURRICULUM CONNECTIONS

None claimed. The lessons in this unit carry their own standards classifications on their own pages, checked lesson by lesson. A unit-level expectation is a different claim, and none has been verified for this unit, so none is printed here. Use the placement guide above to situate the unit in your year and the individual lesson pages for the codes.

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UNIT AT A GLANCE	
# MATHEMATICAL MOVE	SOURCE LESSON COLLECT
1 Exactly twice as long, proved by laying the short one along the long one twice	Pythagoras and the Two strips, the long one built from two Ratios layings of the short, with the proof lesson 1, Exactly performed in front of a partner. Twice as Long
2 Five fours or sixteen more: one pair of heights, two kinds of comparison	Beanstalk: The Two written sentences with their Measure of a Giant equations, one multiplicative and one lesson 1, How Many additive, with the book's own wording Jacks Make a Ray? circled.
3 Exactly twice, or more than twice: which phrase the measurement earns Checkpoint after this lesson	Pythagoras and the A record card with the phrase each Ratios measurement earned and, beside each, lesson 3, Exactly what the strip did. Twice, or More Than Twice
4 Three times, one third, and two hundred sixty more — and the reversal that keeps the wrong word	On the Trail with Three spoken sentences about one pair Lewis and Clark — three times, one third, and two lesson 1, Three hundred sixty more — with the rows Times, or Two pointed at for each. Hundred Sixty More
5 Find the multiplier the book implies, and build the length it demands	Beanstalk: The A strip of painter's tape on the wall at Measure of a Giant the computed height, with the equation lesson 2, Build a written beside the card. Hoop That Is Three Times a Height
6 Use the multiplier to test a claim the book never works out Checkpoint after this lesson	Beanstalk: The The missing factor written and checked, Measure of a Giant and the computed height written before lesson 3, Check the claim is read. Jack's Number

7 Twice as much and half as much, said in words the book never uses	Follow Your Money Two strips with the fold performed, and lesson 10, Twice as the factor and the difference both Much, Half as Much: recorded.
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Saying the Jeans Both Ways

UNIT VOCABULARY

Words students say, not words they are taught to define.

times as tall The multiplicative comparison. Children write it with its equation underneath.

taller than The additive comparison, kept separate from the multiplicative one.

exactly

Lesson 1: the word that makes a comparison a measuring job.

one third of

Lesson 4: the reverse of three times, which the books never print.

MATERIALS

From the source lessons Each day's materials are listed on that day's page below and on the source lesson itself.

Nothing is duplicated here.

New for this unit Checkpoint 1 page, one per child Checkpoint 2 page, one per child Performance task set: adding machine tape, painter's tape, a tape measure, unit cubes and a card naming two lengths, per pair Performance task page, one per child

ASSESSMENT PLAN

Daily evidence Keep the two written comparisons from lesson 2, the chosen phrase with its evidence from

lesson 3, and the three sentences from lesson 4.

Checkpoint 1, after lesson 3: Both comparisons Two measured strips, 6 inches and 18 inches. The child writes the multiplicative comparison with its equation and the additive comparison with its equation.

Book-free. Plain strips, no book.

Look for. Three times as long with $3 \times 6 = 18$, and twelve inches longer with $18 - 6 = 12$.

Both are required; one alone is half the answer.

Checkpoint 2, after lesson 6: Check the claim A shadow of 3 feet belongs to an object 30 feet tall. A second shadow measures 27 feet.

Somebody claims the second object is 260 feet tall. The child finds the multiplier, applies it, and says whether the claim holds.

Book-free. Numbers on a card, no book.

Look for. A multiplier of ten, an answer of 270, and a clear verdict that the claim is wrong by ten.

Culminating task: Say it both ways, and check it

Each pair takes two lengths from their card, builds them as strips of tape on the floor, and lays copies of the short one along the long one to find the factor. They write four sentences about the pair: the factor forwards and backwards, and the difference forwards and backwards, each with its equation. They then use their factor on a third length from the card, build the answer, and finally test a claim written on the card by computing it and marking it agree or disagree.

Collect. The strips on the floor, the four written sentences with their equations, the built third length, and the verdict on the claim.

What it assesses. Both kinds of comparison in both directions, applying a multiplier, and using it to test a claim — the whole of what the unit built.

DAILY LESSON PAGES

Lesson 1 Exactly twice as long, proved by laying the short one along

the long one twice

SKILL LESSON Pythagoras and the Ratios

- lesson 1, Exactly Twice as Long · Grade 4 · 20 minutes Doubling a measured length and checking the double by laying the shorter one along it

I CAN

I can build a length exactly twice another.

BIG IDEA

Exactly is what turns a comparison into a measuring job, and the proof is that the short strip lays along the long one twice with no gap and no overhang.

SUCCESS

The second laying finishes exactly at the far end, and the child can say by how much it misses if it does not.

CONNECT	5 MIN
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First lesson. Hold up two strips and say one is twice the other. Ask how anybody could check, and take the answers before any measuring.

TAUGHT ON ITS OWN

Pythagoras will not tune the pipes by ear; he gives one instruction with a multiplier in it, and the word exactly is what makes it a measuring job.

SOURCE CORE	20 MIN
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Pairs read the instruction and say what would count as following it, then build a second strip exactly twice the short one without a ruler, by laying the short strip down, marking where it ends, and laying it again from that mark.

COLLECT

Two strips, the long one built from two layings of the short, with the proof performed in front of a partner.

CLOSE	5 MIN
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Each pair says one sentence to another pair naming both strips and using the words exactly twice as long, and the other pair checks it by laying the short strip along the long one while they watch.

From the source lesson: The two lines of Pythagoras's instruction on a card, one per pair; An envelope per pair holding one short paper strip and one long blank strip; A pencil; The two strips from the previous step; A report card with one sentence line, one per student; A long strip premarked at its halfway point, for the Support tier; A second long blank strip, for the Extension tier;

Word cards: exactly, twice as long, measure

Watch for: A sentence that could describe either strip. It has to say which one is the long one, or it does not pass.

Lesson 2 Five fours or sixteen more: one pair of heights, two kinds of

comparison

SKILL LESSON Beanstalk: The Measure of a Giant Jacks Make a Ray? · Grade 4 · 20 minutes

Multiplicative comparison of two measured heights

- lesson 1, How Many

I CAN

I can say a comparison two different ways.

BIG IDEA

One pair of heights carries a factor and a difference at the same time, and both are true.

One pair of heights carries a factor and a difference at the same time, and both are true.

SUCCESS

Both sentences are written with their equations underneath, and the child circles the one whose words match the book's.

CONNECT	5 MIN
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Yesterday exactly twice was proved by laying. Today the same laying gives a second number that has nothing to do with the factor.

TAUGHT ON ITS OWN

The book puts two heights side by side and never lets them go: Ray at twenty feet, Jack at four, and every problem comes out of that pair.

SOURCE CORE	20 MIN
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Groups tear tape at four feet and at twenty and label them, then lay four-foot lengths head to tail along the twenty-foot strip counting aloud until it is covered.

COLLECT

Two written sentences with their equations, one multiplicative and one additive, with the book's own wording circled.

CLOSE	5 MIN
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Students take the two hoop heights, sixty feet and twelve, and answer how many of Jack's hoops stack up to Ray's, writing the equation and saying what they notice about the number.

From the source lesson: Tape measure marked in feet; Adding machine tape; Painter's tape;

Lined paper; Pencils; Word cards: equation, measure, taller than, times as tall as

Watch for: A child gives the difference when the factor was asked for, or the reverse. Both sentences are required, every time.

Lesson 3 Exactly twice, or more than twice: which phrase the

measurement earns

SKILL LESSON Pythagoras and the Ratios

- lesson 3, Exactly Twice, or More Than Twice · Grade 4 · 20 minutes Deciding whether a comparison of two measured lengths may be called exact

I CAN

I can say which phrase my measurement earns.

BIG IDEA

Two comparisons of the same kind between the same two objects, and only one of them is allowed the word exactly.

SUCCESS

Each chosen phrase is written with what the strip actually did beside it.

CONNECT	5 MIN
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You have said a comparison two ways. Today one sentence in the book contains two comparisons and gives them different words.

TAUGHT ON ITS OWN

Pythagoras compares the two sets of pipes in one sentence and uses two different comparisons in it: the widths are exactly twice, the lengths are more than twice.

SOURCE CORE	20 MIN
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Pairs mark the small pipe's width on a narrow strip and lay the marked strip across the large pipe twice, saying whether it finishes exactly, falls short or overhangs; then do the same for the two lengths.

COLLECT

A record card with the phrase each measurement earned and, beside each, what the strip did.

CLOSE	5 MIN
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Each pair says its two chosen phrases to another pair with the strips on the table, and the listening pair may not agree until they have laid the marked strip themselves.

From the source lesson: The comparison sentence enlarged on a card, one per pair; An envelope per pair holding two card pipes, one small and one large; A narrow paper strip; A pencil;

A second narrow paper strip; A record card with two lines and a space beside each, one per student; A wider paper strip, for the Extension tier; Word cards: exactly, more than, width, length

Watch for: A pair chooses exactly because the book used it. The strip decides, not the page.

Checkpoint after this lesson: Both comparisons. The page is at the end of this guide.

Lesson 4 Three times, one third, and two hundred sixty more — and the

reversal that keeps the wrong word

SKILL LESSON On the Trail with Lewis and Clark Times, or Two Hundred Sixty More · Grade 4 · 20 minutes Comparing two stated monthly distances additively and multiplicatively

- lesson 1, Three

I CAN

I can catch a reversal that keeps the wrong word.

BIG IDEA

Three times, one third and two hundred sixty more are three true sentences about one pair, and only one word has to change when the direction does.

SUCCESS

The child says which word has to change in the deliberately wrong reversal, and changes it.

CONNECT	5 MIN
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You can choose the phrase a measurement earns. Today the phrase is right and the direction is wrong, which is harder to see.

TAUGHT ON ITS OWN

One sentence carries the highest monthly distance and asks for the lowest, and the answer sits directly after it.

SOURCE CORE	20 MIN
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Pairs write the two months and their miles on strips, build both in base-ten pieces in rows that start level, and find how many times the November row fits inside the September row by laying copies of it end to end.

COLLECT

Three spoken sentences about one pair — three times, one third, and two hundred sixty more — with the rows pointed at for each.

CLOSE	5 MIN
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A partner says one of the three sentences with the two months swapped but the comparison word left alone; the pair decides whether it is still true, says which word has to change, and changes it.

From the source lesson: Printed source passage in this step; Paper strips, two per pair; Baseten pieces, one tray per pair; Lined half-sheet, one per student; Sticky note, two per pair; Word cards: times, third, more

Watch for: A pair accepts the reversed sentence because the number is right. The number is right and the sentence is false; that is the catch.

Lesson 5 Find the multiplier the book implies, and build the length it demands

SKILL LESSON Beanstalk: The Measure of a Giant Hoop That Is Three Times a Height · Grade 4 · 20 minutes Multiplying a whole number by three to build a matching measure

- lesson 2, Build a

I CAN

I can find the multiplier and build what it demands.

BIG IDEA

A multiplier can be recovered from two numbers about one thing and then applied to something else.

SUCCESS

The tape is on the wall at the right height and the child can say how many times their own height it is.

CONNECT	5 MIN
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Yesterday you found a factor between two given numbers. Today the factor is the thing you have to find first, and then use.

TAUGHT ON ITS OWN

Ray gives two numbers about himself and Jack reads a rule out of them.

SOURCE CORE	20 MIN
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Pairs write Ray's height and hoop height on a card, test whether three really carries twenty to sixty by skip counting on a number line and writing the equation, then compute three times four and mark that height on the wall with the tape measure and a strip of tape.

COLLECT

A strip of painter's tape on the wall at the computed height, with the equation written

A strip of painter's tape on the wall at the computed height, with the equation written beside the card.

CLOSE	5 MIN
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Each pair picks a whole number of feet for a new player, works out that player's hoop height, and says the finished sentence to another pair, who check the arithmetic out loud.

From the source lesson: Index cards; Pencils; Chart paper; Markers; Tape measure marked in feet; Painter's tape; Word cards: height, measure, skip count, three times

Watch for: A pair computes and never measures. The tape on the wall is what turns the arithmetic into a length.

Lesson 6 Use the multiplier to test a claim the book never works out

SKILL LESSON Beanstalk: The Measure of a Giant Jack's Number · Grade 4 · 20 minutes Finding a missing factor and multiplying a two-digit number by ten

- lesson 3, Check

I CAN

I can use a multiplier to check a claim.

BIG IDEA

A comparison is a tool: once the factor is known it can be run on a second measurement to test something somebody said.

SUCCESS

The child writes the product before reading the claim and then marks agree or disagree.

CONNECT	5 MIN
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You can find a multiplier and build with it. Today somebody has already shouted an answer and your multiplier is going to check it.

TAUGHT ON ITS OWN

Jack measures two shadows and then shouts a height for the beanstalk that the book never works out on the page.

SOURCE CORE	20 MIN
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Pairs find the two shadow measurements, write two times blank equals twenty and find the missing factor, check it against Ray's stated height, then multiply forty-one by ten showing it as forty times ten plus one times ten.

COLLECT

The missing factor written and checked, and the computed height written before the claim is read.

CLOSE	5 MIN
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Each student answers in writing what height the same ten would give if the shadow had measured forty rather than forty-one, and how far off that is from the claim, then reads the sentence to a partner.

From the source lesson: Index cards; Pencils; Lined paper; Word cards: difference, missing factor, ten times

Watch for: A child reads the claim first and works backwards to it. Cover the claim until the product is written.

Checkpoint after this lesson: Check the claim. The page is at the end of this guide.

Lesson 7 Twice as much and half as much, said in words the book never uses

MATH THROUGH THE READING Follow Your Money

- lesson 10, Twice as Much, Half as Much: Saying the Jeans Both Ways · Grade 4 · 20 minutes
Students build two bar strips for the two jean prices, discover the longer folds exactly onto the shorter, and say the relation in both directions using words the book never uses.

I CAN

I can supply the sentence the book never writes.

BIG IDEA

Forty and eighty say nothing about each other until somebody says twice, and the fold is what produces it.

SUCCESS

The child says both directions of the relation about strips that do not move between the two sentences.

CONNECT	5 MIN
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You have run a multiplier and used it to test a claim. Today two numbers sit side by side in a book that never relates them at all.

TAUGHT ON ITS OWN

This book compares constantly and quantifies both sides every time, and never once says a relation.

SOURCE CORE	20 MIN
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Pairs build two bar strips in ten-dollar units for the two jean prices, fold the longer onto the shorter to test whether it covers exactly twice, and count the units left standing to find the difference.

COLLECT

Two strips with the fold performed, and the factor and the difference both recorded.

CLOSE	5 MIN
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Each student says and writes the relation in both directions using words the book never uses, and a partner checks that the strips did not move between the two sentences.

From the source lesson: Regular jeans and designer jeans ledger cards, one pair per two students; Ten-dollar unit strips, twelve per pair; Single-dollar unit strips, seventy per pair; Shared baseline card, one per pair; Sentence frame card with both directions printed, one per student;

The two jeans tables enlarged with their top lines and last lines highlighted; One pair of built strips shown folded, displayed only after every pair has folded their own

Watch for: A child says twice as much and stops. The second direction is the whole lesson; ask for it explicitly.

CULMINATING PERFORMANCE TASK

Say it both ways, and check it Each pair takes two lengths from their card, builds them as strips of tape on the floor, and lays copies of the short one along the long one to find the factor. They write four sentences about the pair: the factor forwards and backwards, and the difference forwards and backwards, each with its equation. They then use their factor on a third length from the card, build the answer, and finally test a claim written on the card by computing it and marking it agree or disagree.

Collect. The strips on the floor, the four written sentences with their equations, the built third length, and the verdict on the claim.

What it assesses. Both kinds of comparison in both directions, applying a multiplier, and using it to test a claim — the whole of what the unit built.

TEACHER KEY AND RUBRIC

Key Checkpoint 1: three times as long, $3 \times 6 = 18$; twelve inches longer, $18 - 6 = 12$.

Checkpoint 2: the multiplier is ten; $27 \times 10 = 270$; the claim of 260 is wrong by ten.

Task: choose two lengths where one is a whole-number multiple of the other, and write the claim so that it is wrong by a small amount. Work all four sentences and the claim yourself before the lesson.

Rubric		
STRAND SECURE LOOKS LIKE	DEVELOPING	NOT YET
Representation The factor is found by laying copies of the short length along the long one, not by dividing on paper first.	Does it with the arrangement in front of them.	Produces an answer without the arrangement or the reason.
Calculation Every sentence carries its equation and the equations are correct.	Does it with the arrangement in front of them.	Produces an answer without the arrangement or the reason.
Meaning The multiplicative and additive comparisons are kept distinct and both are stated.	Does it with the arrangement in front of them.	Produces an answer without the arrangement or the reason.
Explanation The verdict on the claim names the number the child computed and the number the claim gave.	Does it with the arrangement in front of them.	Produces an answer without the arrangement or the reason.

CHECKPOINT 1 · GRADE 4 Both comparisons	
Name	Date

Two strips, 6 inches and 18 inches. Write the comparison using times, with its equation underneath. Now write the comparison using longer than, with its equation.

CHECKPOINT 2 · GRADE 4 Check the claim	
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Name	Date
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A shadow of 3 feet belongs to something 30 feet tall. A second shadow measures 27 feet. Somebody says the second thing is 260 feet tall. Find the multiplier, use it, and say whether the claim holds.

CULMINATING TASK · GRADE 4 Say it both ways, and check it	
Name	Date

Build the two lengths from your card as strips on the floor. Lay copies of the short one along the long one to find how many times it fits. Write four sentences about the pair, each with its equation: the factor forwards and backwards, and the difference forwards and backwards. Now use your factor on the third length and build the answer. Last, work out the claim on your card and mark it agree or disagree.

TEACHER PLANNING AND HANDOFF

Children who gave only one comparison at checkpoint 1 need lesson 2 again before any ratio work.

Lesson 3's exactly is the piece that carries into measurement, where a claim of exactness has to be earned.

Beanstalk supplies three lessons because it is the only Grade 4 book that names both comparisons, applies a multiplier and uses one to test a claim.

TumbleTA - Tumbleweed Press Inc. - Unit built 2026-09-02 from lessons published under Math lessons.

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