

# Where This Fits in Your Year

EQUAL PARTS OF ONE WHOLE · GRADE 3 MATHEMATICS · PENNSYLVANIA

Natural unit: Numbers and Operations - Fractions This five-lesson unit fits most naturally in the first sustained Grade 3 work on fractions as numbers. It is best used after students can partition familiar shapes informally but before fraction procedures become the focus. Lesson 1 establishes that the number of pieces is not enough: the pieces must be equal before names such as thirds or sixths apply. Lesson 2 then compares unit fractions made from same-size wholes, so students can see why more equal parts produce a smaller part.

Lessons 3 and 4 deepen the part-whole relationship. Students prove  $2/8 = 1/4$  and  $6/8 = 3/4$  by covering, then build one whole as  $4/4$  before removing  $1/4$  and recording  $4/4 - 1/4 = 3/4$ . The models come before the equations, making the symbols records of actions rather than isolated rules.

Lesson 5 asks the question that keeps fraction language precise: one eighth of what? Students compare sharing one cup among eight pots with sharing eight cups among the same eight pots. The unit is strongest as a focused bridge into equivalence, comparison, and reasoning about the referent whole. It does not replace later work on number lines, broader equivalence, or the full regional fraction progression.

## STRONGEST CURRICULUM CONNECTIONS - OUR SUMMARY

No coded connection is printed because a current compact code and wording were not directly verified. The instructional placement above remains usable and honest.

### Teacher must add

- Select the applicable current Grade 3 fraction expectation locally. No compact code is printed because an exact current code and wording were not directly verified.

Official source: Pennsylvania Academic Standards for Mathematics, Grade 3 Placement describes what students explicitly do in this unit; it is not a claim that the complete resource fully aligns to the curriculum.

EQUAL PARTS OF ONE WHOLE

## Equal Parts of One Whole

GRADE 3 MATHEMATICS UNIT

### UNIT PROMISE

Five focused lessons move from equal shares, to unit-fraction size, equivalence, one whole as  $n/n$ , and finally naming a fraction in relation to its whole. Students cut, stack, measure, explain, and revise.

# Where This Fits in Your Year

Use this compact unit after students can partition familiar shapes informally and before they add, subtract, or place fractions on a number line. It establishes the language and reasoning those later ideas depend on: equal parts must come from the same whole, the denominator names how many equal parts make the whole, and more equal parts means each part is smaller.

The sequence is intentionally coherent. Lesson 1 protects the meaning of equal. Lesson 2 compares unit fractions only when the wholes are the same size. Lessons 3 and 4 use covering and measuring to establish equivalence and one whole as  $n/n$ . Lesson 5 separates a fraction of one whole from an equal-share division result.

|                     |                                                                                                              |
|---------------------|--------------------------------------------------------------------------------------------------------------|
| <b>LENGTH</b>       | 5 lessons · 30 minutes each                                                                                  |
| <b>STUDENT</b>      | Two checkpoints, daily evidence, and one culminating task                                                    |
| <b>PRODUCTS</b>     |                                                                                                              |
| <b>SOURCE BOOKS</b> | Pizza Counting; Manga Math Mysteries #5: The Ancient Formula                                                 |
| <b>MATERIALS</b>    | Printed circles, scissors, crayons, dry rice, fraction measures, eight small cups or pots, water or counters |
| <b>PREREQUISITE</b> | Students can count equal groups and use halves and fourths informally                                        |

## MATH THROUGH READING

Each lesson opens with a short claim that students must interpret precisely. The mathematics changes when one word changes: parts becomes equal parts; an eighth becomes an eighth of one cup. Students learn to treat mathematical language as evidence.

|                                                                                                                    |                                           |
|--------------------------------------------------------------------------------------------------------------------|-------------------------------------------|
| Unit at a Glance                                                                                                   |                                           |
| Essential question How does the whole determine the name and size of a fractional part?                            |                                           |
| LESSON MATHEMATICAL TURN                                                                                           | STUDENT EVIDENCE                          |
| 1 · May it be called thirds? Three pieces are not necessarily thirds; all three must be equal shares of one whole. | Cut, stack, and justify a classification. |

|                                                                                                                                         |                                                          |
|-----------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------|
| 2 · Which piece is bigger? For same-size wholes, sixths are larger than eighths because fewer equal parts compose the whole.            | Compare one sixth and one eighth with a model and words. |
| 3 · Two names, same Two eighths and one fourth cover the same area, so $2/8$ amount = $1/4$ .                                           | Build and record an equivalence by covering.             |
| 4 · Take one fourth away Four fourths name one whole; removing one fourth leaves three fourths.                                         | Measure and record $4/4 - 1/4 = 3/4$ .                   |
| 5 · What is it a part of? A fraction name is incomplete without identifying the whole; equal sharing can produce a whole-number amount. | Explain two different eight-way shares.                  |

## Assessment lens

Look for five non-negotiable ideas: (1) equal parts, not merely the correct number of parts; (2) comparisons made from same-size wholes; (3) equivalence justified by covering or measuring; (4) one whole named as  $n/n$  before a fractional part is removed; and (5) a named referent for every fraction. A correct label or equation without its model and conditions is not yet secure understanding.

## Vocabulary

|                       |                                                                                                                 |
|-----------------------|-----------------------------------------------------------------------------------------------------------------|
| <b>WORDS TO TEACH</b> | whole · equal parts · third · fourth · sixth · eighth · equivalent · unit fraction · share                      |
| <b>SENTENCE</b>       | These may/may not be called ____ because ____ . · One ____ is larger than one ____ because ____ . · This is one |
| <b>FRAMES</b>         | eighth of ____.                                                                                                 |

### COMMON MISCONCEPTION

A larger denominator does not mean a larger piece. The denominator counts equal parts in the whole; when the whole stays fixed, dividing it into more equal parts makes each part smaller.

## Lesson 1 · May It Be Called Thirds?

### EQUAL PARTS DEFINE THE FRACTION

|                  |                                                                             |
|------------------|-----------------------------------------------------------------------------|
| <b>SOURCE</b>    | Pizza Counting · Lesson 2, Cut Into Six, Divided Into Sixths                |
| <b>OBJECTIVE</b> | Decide whether a whole is partitioned into thirds and justify the decision. |
| <b>STUDENT</b>   | A cut-and-stack comparison with a written or dictated reason.               |
| <b>PRODUCT</b>   |                                                                             |
| <b>TIME</b>      | 30 minutes                                                                  |
| <b>MATERIALS</b> | Checkpoint 1, scissors, crayons                                             |

## Lesson sequence

1. Read the claim · 4 min - Display: Both circles have three pieces, so both circles show thirds. Ask students to identify the word that needs testing.
2. Build the definition · 6 min - Establish that thirds are three equal parts of one whole. Count alone cannot prove the name.
3. Cut and stack · 8 min - Students cut the pieces from each circle and stack pieces from the same circle. They mark the circle whose pieces match.
4. Explain and challenge · 7 min - Partners use: These may/may not be called thirds because... Ask whether rotating a piece changes its size.
5. Close · 5 min - Change the original claim so it is always true.

### WATCH FOR

Students may say that pieces are equal because they look similar. Require physical alignment or a clear symmetry argument. Do not accept “three pieces” as sufficient evidence.

### CHECKPOINT

A secure response identifies Circle A and states that its three parts are equal. Circle B has three parts but not thirds.

### EXTENSION

Draw a quadrilateral split into three equal areas in a way that does not create three congruent shapes. Discuss equal area versus identical appearance.

# Checkpoint 1 · May It Be Called Thirds?

Name \_\_\_\_\_ Date \_\_\_\_\_ Here are two same-size circles, each cut into three pieces. Cut out each circle. Stack or align the pieces from the same circle.

Which circle may be called thirds? Explain how you know.

Circle A

Circle \_\_\_\_\_ may be called thirds because

Circle B

---

## CHECK YOUR WORDS

Did you name the whole, the number of parts, and whether the parts are equal?

## Lesson 2 · Which Piece Is Bigger?

COMPARE UNIT FRACTIONS FROM THE SAME WHOLE

|                  |                                                                                        |
|------------------|----------------------------------------------------------------------------------------|
| <b>SOURCE</b>    | Pizza Counting · Lesson 1, The Slice That Looks Smaller Because There Are More of Them |
| <b>OBJECTIVE</b> | Compare one sixth and one eighth when the wholes are the same size.                    |
| <b>STUDENT</b>   | A model-based comparison and denominator explanation.                                  |
| <b>PRODUCT</b>   |                                                                                        |
| <b>TIME</b>      | 30 minutes                                                                             |
| <b>MATERIALS</b> | Practice page, scissors, two crayons                                                   |

## Lesson sequence

1. Read two claims · 4 min - Compare: Eight is greater than six and One eighth is greater than one sixth. Ask why the first can be true while the second is false here.
2. Fix the whole · 5 min - Place two same-size circles together. Emphasize that fraction-size comparison is only justified because the wholes match.

3. Partition and compare · 9 min - Students cut one sixth and one eighth, then stack them. They record  $1/6 > 1/8$ .
4. Reason without recounting · 7 min - Ask: If the same whole is shared among more people, does each person receive more or less?

Connect the share count to the denominator.

5. Close · 5 min - Students finish: For the same whole, more equal parts means...

### WATCH FOR

Students may compare the numerals 6 and 8 rather than the pieces. Return to the action: more equal shares of the same amount means a smaller share.

### CHECKPOINT

The student selects one sixth, writes  $1/6 > 1/8$ , and explicitly cites same-size wholes or more equal parts.

### EXTENSION

Predict and then test one fourth versus one sixth. Ask students to generalize cautiously to unit fractions only.

## Practice Page · Which Piece Is Bigger?

Name \_\_\_\_\_ Date \_\_\_\_\_ The circles started the same size. Cut out one sixth and one eighth. Stack the pieces. Circle the larger piece and explain why it is larger.

| Sixths                                  | Eighths |
|-----------------------------------------|---------|
| Write the comparison: $1/6$ _____ $1/8$ |         |
| One _____ is larger because             |         |

### IMPORTANT CONDITION

Your explanation must say that the two wholes started the same size.

## Lesson 3 · Two Names for the Same Amount

### PROVE EQUIVALENCE BY COVERING

|                  |                                                             |
|------------------|-------------------------------------------------------------|
| <b>SOURCE</b>    | Manga Math Mysteries #5: The Ancient Formula · Lesson 2     |
| <b>OBJECTIVE</b> | Show that two eighths and one fourth name the same amount.  |
| <b>STUDENT</b>   | A covered model and the equation $2/8 = 1/4$ .              |
| <b>PRODUCT</b>   |                                                             |
| <b>TIME</b>      | 30 minutes                                                  |
| <b>MATERIALS</b> | Paper disks, scissors, fourth-circle template, record strip |

## Lesson sequence

1. Reconnect · 4 min - Yesterday pieces were compared from same-size wholes. Today two fraction names will describe the same amount.
2. Build eighths · 7 min - Fold a disk in half three times, cut eight wedges, and stack them to check that they match.
3. Cover a fourth · 8 min - Place two eighth wedges over one fourth wedge. Align points and edges; accept equivalence only when no paper extends beyond the other shape.
4. Record the proof · 6 min - Trace the covered shape and write  $2/8 = 1/4$  beside the model.
5. Close · 5 min - Build six eighths, group them in pairs against the fourth template, and record  $6/8 = 3/4$ .

### WATCH FOR

Approximate wedges make approximate evidence. If pieces do not match, fold a fresh disk instead of trimming a piece to force the result.

### CHECKPOINT

The model completely covers, and the equation matches the model. The equals sign is justified by covering, not by a memorized rule.

### EXTENSION

Predict how many eighths will cover two fourths, then verify physically.

## Lesson 4 · Take One Fourth Away

NAME THE WHOLE AS FOUR FOURTHS BEFORE SUBTRACTING

|               |                                                         |
|---------------|---------------------------------------------------------|
| <b>SOURCE</b> | Manga Math Mysteries #5: The Ancient Formula · Lesson 1 |
|---------------|---------------------------------------------------------|

|                  |                                                                          |
|------------------|--------------------------------------------------------------------------|
| <b>OBJECTIVE</b> | Represent one whole as $\frac{4}{4}$ and subtract one fourth.            |
| <b>STUDENT</b>   | A measured model and $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$ .         |
| <b>PRODUCT</b>   |                                                                          |
| <b>TIME</b>      | 30 minutes                                                               |
| <b>MATERIALS</b> | Four one-fourth measures, one-cup measure, dry rice, labels, recipe card |

## Lesson sequence

1. Reconnect · 4 min - Yesterday one amount had two fraction names. Today the whole receives a fraction name so one part can be removed.
2. Build the whole · 7 min - Pour four one-fourth measures into one cup. Count each pour and state: four fourths is one whole.
3. Remove one part · 7 min - Set one fourth aside as the blue part. Count the three fourths remaining as the red part.
4. Record · 7 min - Write  $1 = \frac{4}{4}$  and  $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$ . Point to the measured quantity named by each number.
5. Close · 5 min - Rebuild the three-fourths remainder and verify it with a three-fourths measure.

### WATCH FOR

Do not begin with symbols. The four one-fourth pours establish the whole as a countable fractional amount; the equation records that action.

### CHECKPOINT

The student writes both equations correctly and identifies the one fourth removed and the three fourths remaining.

### EXTENSION

Remove two fourths instead. Record the result in fourths, then connect it to Lesson 3's equivalence.

## Lesson 3-4 · Recording Tools

Name \_\_\_\_\_ Date \_\_\_\_\_ Lesson 3 record strip · Build each equality with fraction pieces. Trace or sketch the covering, then complete the equation.

## EQUATION

## Lesson 4 recipe card · Record what the measures prove.

## EVIDENCE CHECK

## Checkpoint 2 · Same Amount, One Whole

page 9

## Lesson 5 · What Is It a Part Of?

NAME THE REFERENT; DISTINGUISH FRACTIONS FROM QUOTIENTS

|                  |                                                                                       |
|------------------|---------------------------------------------------------------------------------------|
| <b>SOURCE</b>    | Manga Math Mysteries #5: The Ancient Formula · Lesson 5                               |
| <b>OBJECTIVE</b> | Name unit fractions in relation to a whole and explain two eight-way sharing results. |
| <b>STUDENT</b>   | A record of two sharings and a complete referent sentence.                            |
| <b>PRODUCT</b>   |                                                                                       |
| <b>TIME</b>      | 30 minutes                                                                            |
| <b>MATERIALS</b> | One-cup measure, eight pots/cups, water or counters, recording page                   |

### Lesson sequence

1. Read the incomplete sentence · 4 min - Display: This amount is one eighth. Ask: one eighth of what? A fraction names a relationship to a whole.
2. Share one whole · 7 min - Share one cup equally among eight pots. Each receives  $\frac{1}{8}$  cup. Name both the amount and its referent.
3. Share eight wholes · 7 min - Share eight cups among the same eight pots. Each receives 1 cup-not  $\frac{1}{8}$  cup. Record  $8 \div 8 = 1$ .
4. Compare the sentences · 7 min - Read both results aloud. Circle the words that name the original quantity and underline the amount in each pot.
5. Close · 5 min - Complete: One eighth of \_\_\_\_ is \_\_\_\_ for the first sharing and explain why it does not describe the second.

### WATCH FOR

Students may answer one eighth whenever they see eight recipients. Ask what total amount is being shared. The number of groups alone does not determine the amount in each group.

### CHECKPOINT

Students distinguish  $1 \text{ cup} \div 8$  from  $8 \text{ cups} \div 8$  and name the referent whole in the fractional result.

### EXTENSION

Write two different sharing stories whose answers are  $\frac{1}{8}$  and 1.

# Culminating Task · Build and Name the Parts

Name \_\_\_\_\_ Date \_\_\_\_\_

1. Partition Circle A into four equal parts. Partition Circle B into eight equal parts. Shade one part in each circle and label each unit fraction.

Circle A shows \_\_\_\_\_ of the whole. Circle B shows \_\_\_\_\_ of the whole.

2. The circles are the same size. Which shaded part is larger? Explain using the number of equal parts-not just the numerals.

---

---

3. Complete the naming sentence.

The shaded piece is one \_\_\_\_\_ of \_\_\_\_\_.

4. Use your models to complete:  $\frac{2}{8} =$  \_\_\_\_\_ and  $\frac{6}{8} =$  \_\_\_\_\_

# Culminating Task · Share the Whole

Name \_\_\_\_\_ Date \_\_\_\_\_ Use eight pots or the drawings below.

| 1 2 3 4 5 6                                                                          | 7 | 8 |
|--------------------------------------------------------------------------------------|---|---|
| 1. Share one cup equally among eight pots.                                           |   |   |
| Each pot receives _____ cup. Equation: $1 \div 8 =$ _____                            |   |   |
| 2. Share eight cups equally among the same eight pots.                               |   |   |
| Each pot receives _____ cup(s). Equation: $8 \div 8 =$ _____                         |   |   |
| 3. Explain why the answers are different even though both situations use eight pots. |   |   |

---

---

---

## FINAL SENTENCE

One eighth means one of eight equal parts of a named whole. Write one example:

---

4. A whole is four fourths. One fourth is removed. Complete:  $\frac{4}{4} - \frac{1}{4} =$  \_\_\_\_\_

# Teacher Assessment Guide

|                      |                                                                                                                                                                          |
|----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>CHECKPOINT 1</b>  | Circle A. Its three pieces are equal, so each is one third of the circle.<br>Circle B has three unequal pieces and does not show thirds.                                 |
| <b>PRACTICE PAGE</b> | $\frac{1}{6} > \frac{1}{8}$ . Same-size wholes cut into fewer equal parts produce larger unit fractions.                                                                 |
| <b>CHECKPOINT 2</b>  | $\frac{2}{8} = \frac{1}{4}$ ; one whole = four fourths = $\frac{4}{4}$ ; $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$ .                                                     |
| <b>CULMINATING</b>   | $\frac{1}{4}$ is larger than $\frac{1}{8}$ for same-size wholes; $\frac{2}{8} = \frac{1}{4}$ ; $\frac{6}{8} = \frac{3}{4}$ ; $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$ . |
| <b>MODELS</b>        |                                                                                                                                                                          |
| <b>SHARING</b>       | $1 \div 8 = \frac{1}{8}$ cup per pot. $8 \div 8 = 1$ cup per pot.                                                                                                        |

## Three-level rubric

| LEVEL             | EVIDENCE                                                                                                                                                                                                            |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Secure</b>     | Builds equal parts; holds the whole constant; proves equivalence by covering; names one whole as $\frac{n}{n}$ before subtracting; and names the referent in both sharing results. Equations match the models.      |
| <b>Developing</b> | Produces mostly correct models and equations but omits or cannot explain one condition: equality, same-size wholes, equivalent coverage, the whole as $\frac{n}{n}$ , or the named referent.                        |
| <b>Beginning</b>  | Counts pieces without checking equality, compares denominators as sizes, asserts equivalence without evidence, subtracts before building the whole, or answers $\frac{1}{8}$ solely because there are eight groups. |

## INSTRUCTIONAL DECISION

If students miss equality, repeat Lesson 1 with paper strips. If they miss the same-whole condition, contrast different-size wholes. If equivalence is unsupported, return to physical covering. If subtraction is symbolic only, rebuild  $\frac{4}{4}$  with measures before removing  $\frac{1}{4}$ . If they confuse  $1 \div 8$  with  $8 \div 8$ , distribute the quantities again.

## Why this unit is intentionally short

The unit teaches a focused conceptual foundation well. It does not claim to cover all Grade 3 fraction work. Follow it with number-line placement, additional equivalent fractions, whole-number fractions, and broader comparison tasks as required by the local curriculum.