

Compare and Model Fractions

Grade 3 · Mathematics · Where This Fits in Your Year Use this pack during Grade 3 work on fractions, equal sharing, and early multiplication/division. Students partition same-size pizza models into equal parts, compare the size of unit fractions, test whether unequal pieces can legitimately be named as thirds, and model grouping and sharing with concrete slices.

STRONGEST CURRICULUM CONNECTIONS - OUR SUMMARY

TEACHES · 3.NF.A.1

Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

Student evidence: Students cut same-size wholes into equal parts, test equality by stacking pieces, reject an unequal partition as not nameable in thirds, and connect the number of equal parts to fraction names.

Curriculum source: Vermont-adopted Common Core State Standards for Mathematics
<https://education.vermont.gov/>

LESSON GUIDE

Seven Lessons from Compare and Model Fractions

Choose the lesson that fits the work already underway. Each lesson includes a full version, a short version, three levels of access, language support and materials.

1	The Slice That Looks Smaller Because There Are More of Them	Skill
2	Ten Slices, Two a Day, and the Same Ten Again	Skill
3	The Word That Has to Be True Before You Can Say Thirds	Math Through The Reading
4	The Person the Sentence Forgot to Count	Math Through The Reading

5	Talking Someone Out of the Bigger Number	Math Through The Reading
6	The Cat, the Whale, and the Apple Pie	Reading And Language
7	A Pizza Like a Blank Page	Reading And Language

SKILL LESSON

1. The Slice That Looks Smaller Because There Are

More of Them

Concept · Comparing two fractions with the same numerator by reasoning about the size of the parts The book ends by asking a child which of two slices they would rather have, and every one of its four pairs has the same number on top and different numbers underneath. That is the exact case where children answer wrong, because the larger bottom number looks like the larger amount. Students cut circles of one size into thirds and fourths, then into sixths and eighths, stack one piece from each, and see the piece from the more-divided pizza sit inside the other. Concept here means the child can say why more slices makes each slice smaller, with the two pieces in hand as the reason, rather than reciting which fraction is bigger.

Full lesson - 20 minutes

1. Cut the two pizzas the book names (5 min) Each pair takes two paper circles of exactly the same size and folds one into three matching parts and the other into four. Cut on the folds, then stack one third on top of one fourth so their points meet at the centre.

Say which piece hangs out past the other, and keep both pieces on the desk.

Word preview: thirds, fourths A pizza that is cut into three equal slices is divided into thirds.

A pizza that is cut into four equal slices is divided into fourths.

2. Answer the book's own question with the pieces (5 min) The teacher reads the book's question aloud and each pair holds up the piece they would rather have. Say the answer as a whole sentence naming both fractions and the reason, with both pieces still touching. The rule is that the pieces on the desk decide the answer, not the numbers alone.

Word preview: rather, size Which size slice would you rather have? One third or one fourth?

3. Test whether the pattern holds for sixths and eighths (5 min) Each pair folds two more circles of the same size into six parts and into eight, cuts one piece from each, and stacks them the way they stacked the first two. Predict which piece is larger before cutting and write the prediction down.

Then check it and say whether the bigger bottom number gave the bigger piece or the smaller one.

Word preview: sixths, eighths One sixth or one eighth?

4. Say the rule the four pieces show (5 min) Each pair lines up their four pieces from largest to smallest and reads the bottom numbers aloud in the order the pieces sit. State in one sentence what happens to the size of a slice when a pizza of the same size is cut into more slices. Two pairs compare sentences and settle any disagreement by restacking the pieces.

Word preview: equal, whole Pizza fractions: halves thirds, fourths, fifths, sixths, eighths, tenths, twelfths, one whole!

Short version - 9 minutes

1. Cut the two pizzas the book names (4 min) Each pair folds one paper circle into three matching parts and another of the same size into four, then cuts on the folds. Stack one third on one fourth with their points at the centre and say which one hangs out past the other.

Word preview: thirds, fourths A pizza that is cut into three equal slices is divided into thirds. A pizza that is cut into four equal slices is divided into fourths.

2. Answer the book's own question with the pieces (5 min) The teacher reads the book's question aloud and each pair holds up the piece they would rather have. Say the answer as a full sentence naming both fractions and the reason, with the two pieces still stacked.

Word preview: rather, size Which size slice would you rather have? One third or one fourth?

Three Levels

Support

Stay with thirds and fourths only, and use circles the teacher has already folded so the cutting is the child's whole job. Stack the two pieces and answer the book's question by pointing at the piece that hangs out, saying only 'one third is bigger'. Add the sixths and eighths pair only when the first stack has been explained aloud once.

At Grade

Cut all four pizzas, predict before each stack, and say the rule in a sentence that names what happened to the size when the number of slices went up. Both fractions must appear in the sentence.

Extension

Take the book's third question, Two thirds or two sixths, where the top number is two rather than one, and decide it with pieces. Then say whether the rule found for one piece still holds when two pieces are taken, and show it with the pieces rather than asserting it.

English Learners The ordinal words the mathematics rides on are the hard transfer here: third and fourth are the same forms English uses for places in a line, and a child who has met third only as a position may hear the book's question as being about order rather than size. Sixth and eighth carry consonant clusters that are hard to say and easy to lose, and eighth in particular is spelled with letters it does not pronounce. Rehearse the frame 'One ____ is bigger than one ____ because the pizza was cut into fewer slices' with the two pieces held up on the two blanks, so the sentence is anchored to objects and not to sound. Invite students to say which piece is bigger in their strongest language first and then place the two English fraction words into the rehearsed frame; the comparison itself transfers even when the ordinal words do not.

Materials

Paper circles, two of the same size per pair · Scissors · Glue stick · Recording sheet ruled into four rows · Pencils · Word cards: thirds, fourths, sixths, eighths, equal

SKILL LESSON

2. Ten Slices, Two a Day, and the Same Ten Again Application · Partitioning a count of equal slices into equal daily groups and counting the groups The train pizza is the one place this book states a full situation twice over the same ten slices: two a day lasts five days, five a day lasts two days. Students lay out ten counters, group them by two and count the groups, then rebuild the identical ten and group them by five, so the pair of answers comes out of one unchanged set of objects. The calendar pizza repeats the move with twelve and three against the four seasons the book names. Application here means the child uses grouping to answer a question the book asks about eating, and can say why the days went down when the daily slices went up without treating it as two unrelated facts.

Full lesson - 20 minutes

1. Lay out the train pizza's ten slices (5 min) Each pair sets out ten counters in a row as the ten slices of the train pizza and reads the book's sentence naming how it is divided. Push the counters into groups of two, one group for each day, and count the groups aloud. Say how many days the pizza lasted and keep the groups on the desk.

Word preview: tenths, slices This train pizza is divided into tenths. If you ate two slices each day, it would last for five days.

2. Regroup the same ten slices the other way (5 min) Without adding or taking away a counter, the same pair pushes the ten back together and makes groups of five instead. Count the new

groups and say how many days the pizza lasts now. State in one sentence what stayed the same and what changed.

Word preview: groups, days If you're really hungry, you could eat five slices for two days.

3. Do the calendar pizza the same way (5 min) Each pair sets out twelve counters for the calendar pizza and groups them by three, the number of pieces the book says makes one season. Count the groups and check the answer against the four seasons the book names. Say whether the count of groups matches the seasons.

Word preview: twelfths, season This calendar pizza is divided into 12th. If you eat three pieces, you could eat an entire summer, winter, fall, or spring.

4. Say the pattern the two pizzas share (5 min) Each pair writes one sentence saying what happens to the number of days when the number of slices eaten each day goes up, using both of the train pizza's answers as the evidence. Two pairs swap sheets and check the other's sentence against the counters. Restack the counters if the sentences disagree.

Word preview: equal, total The year is divided into 12 months.

Short version - 9 minutes

1. Lay out the train pizza's ten slices (4 min) Each pair sets out ten counters as the slices of the train pizza and reads the book's sentence about it. Push them into groups of two, one for each day, count the groups aloud, and say how many days the pizza lasted.

Word preview: tenths, slices This train pizza is divided into tenths. If you ate two slices each day, it would last for five days.

2. Regroup the same ten slices the other way (5 min) Without adding or taking away a counter, the pair pushes the ten back together and makes groups of five instead. Count the new groups, say how many days it lasts now, and say what stayed the same.

Word preview: groups, days If you're really hungry, you could eat five slices for two days.

Three Levels

Support

Stay with the train pizza's ten counters for the whole lesson and use paper plates so each day's group has a place to sit. Group by two first, count the plates, and only then rebuild and group by five. Say the two answers aloud without writing the comparison sentence.

At Grade

Do both train groupings and the calendar pizza, then write the comparison sentence naming both of the train pizza's answers as the evidence for it.

Extension

Take the book's sixths pizza shared with two friends at two pieces each and decide whether the same grouping move answers it, then say which of the three pizzas asks a different question from the other two and what makes it different.

English Learners The word day is doing double duty in these sentences and that is the trap: it names both the group and the count of groups, so 'two slices each day' and 'it would last for five days' put the same word on two different quantities. A student can group perfectly and still report the wrong number because both answers are days.

Rehearse the frame 'Each day I eat ____ slices, so it lasts ____ days' with a hand on the plate for the first blank and a sweep across all the plates for the second, so the two uses are separated by gesture before they are separated by words. Invite students to say the grouping in their strongest language first, where the two roles may take different words entirely, and then map that onto the English frame.

Materials

Counters, ten per pair · Paper plates, five per pair · Counters, twelve per pair · Recording sheet with two rows · Pencils · Word cards: tenths, twelfths, groups, equal

MATH THROUGH THE READING

3. The Word That Has to Be True Before You Can

Say Thirds

Integrated · Students cut identical pizzas with and without the book's word equal, stack the resulting pieces to see which cuts survive, and decide from the pieces alone which of two circles has earned the name the book gives it.

Students cut identical pizzas with and without the book's word equal, stack the resulting pieces to see which cuts survive, and decide from the pieces alone which of two circles has earned the name the book gives it.

Full lesson - 20 minutes

1. Cut it both ways (6 min) Each pair takes two identical paper circles. On the first they cut three parts any way they like; on the second they fold carefully so the three parts match, then cut. Both sets of pieces stay on the desk, kept apart.
2. Stack and sort (5 min) Pairs stack the three pieces from each circle and sort the two sets into one that stacks with no overhang and one that does not. They say aloud which circle may be called thirds.

3. Find the word that did it (5 min) The teacher reads the book's thirds sentence, then reads it again with equal removed. Pairs point at the word that changed and say which of their two circles each version of the sentence would have allowed.
4. Say the condition (4 min) Each pair states in one sentence what has to be true about the pieces before the naming word may be used, then checks another pair's unequal circle against that statement.

Short version - 9 minutes

1. Cut it both ways (6 min) Each pair takes two identical paper circles. On the first they cut three parts any way they like; on the second they fold carefully so the three parts match, then cut. Both sets of pieces stay on the desk, kept apart.

2. Regroup the same ten slices the other way (5 min)

2. Find the word that did it (5 min) The teacher reads the book's thirds sentence, then reads it again with equal removed. Pairs point at the word that changed and say which of their two circles each version of the sentence would have allowed.

Three Levels

Support

Work with halves rather than thirds so the fold is a single crease and the stack test is unambiguous.

The teacher reads both versions of the sentence twice and the pair points at the changed word rather than naming it. The child says only which circle is allowed, and the reason is supplied by holding up the stacked pair.

At Grade Cut both circles into thirds, run the stack test unaided, and state the condition in a sentence that uses the word equal and refers to the pieces rather than to the count.

Extension

Take the book's sixths sentence and its sharing sentence together, and decide whether a pizza can satisfy cut into six equal slices while failing to give two pieces each to three people. Show the answer with pieces and say which of the two words, equal or each, the failure violates.

English Learners Equal is a false friend in mathematics for many children because its everyday sense is about fairness and its mathematical sense here is about size, and a pair of pieces can be fair to share and still not be equal.

Rehearse the frame 'These pieces are equal because they stack with nothing hanging over' with the stacked pieces in hand, so the word is anchored to the test rather than to the idea of fairness. The naming words are irregular in English, with halves and thirds unpredictable from two and three, so accept the mathematics whenever the condition is stated correctly even if the naming word is built

regularly. Invite students to say what makes a cut fair in their strongest language first and then to say whether that word means the same as equal here; the distinction is often clearer in the first language than in English.

Materials

Paper circles, two of the same size per pair · Scissors · The book's thirds sentence written out with the word equal on a card that can be · lifted away

MATH THROUGH THE READING

4. The Person the Sentence Forgot to Count Integrated · Students deal out the six slices of the book's target pizza at two per person to find how many people are eating, then check that number against the party the book names and explain why the reader has to be counted too.

At Grade

Students deal out the six slices of the book's target pizza at two per person to find how many people are eating, then check that number against the party the book names and explain why the reader has to be counted too.

Full lesson - 20 minutes

1. Deal the slices (5 min) Each pair cuts a paper circle into the six equal slices the book gives the target pizza, then deals them onto plates at two slices a plate until the slices run out. They count the plates and write that number down without saying it aloud.
2. Find the sentence that says how many (6 min) The teacher rereads the sharing sentence twice.

Pairs decide how many people are at this pizza according to the words alone, point at the part of the sentence they used, and say their number.

3. Put the two answers together (5 min) Pairs compare the number of plates with the number of people they read out of the sentence. Where a pair read two people, they set out two plates and say what happens to the slices left over.
4. Say who got missed (4 min) Each pair states in one sentence who the phrase two friends leaves out and why that person still eats, using the plates as the evidence.

Short version - 9 minutes

1. Deal the slices (5 min) Each pair cuts a paper circle into the six equal slices the book gives the target pizza, then deals them onto plates at two slices a plate until the slices run out. They count the plates and write that number down without saying it aloud.
2. Find the sentence that says how many (6 min) The teacher rereads the sharing sentence twice.

Pairs decide how many people are at this pizza according to the words alone, point at the part of the sentence they used, and say their number.

Three Levels

Support

Use six slices already cut and plates already on the desk so the whole task is the dealing. The teacher rereads the sharing sentence a third time with a hand raised on the word share, and the pair answers by putting out plates rather than by saying a number. The leftover pair of slices in the two-plate arrangement is pointed at rather than explained.

At Grade Deal the six unaided, read the number of people out of the sentence before comparing, and state in one sentence who the phrase two friends leaves out.

Extension

Take the calendar pizza, where the book states twelve pieces and says three pieces make a season but never says how many seasons there are. Find that number the same way and say which of the two problems

At Grade

gave more help, then name what the calendar sentence supplies that the sharing sentence does not.

English Learners The sentence turns on whether the speaker is inside or outside the group being named, and languages differ sharply here: several mark inclusive and exclusive we with different words, so a student whose first language makes that distinction grammatically may find this problem easier than the English does and may be puzzled that English leaves it to inference. Say so directly and treat it as a place where English is the less precise language. Rehearse the frame 'Two friends and me makes ___ people, so each of us gets ___ slices' with a hand on the chest for the middle word. The word each carries the per-person reading and is easily heard as a total, so pair it with the dealing gesture every time it is said.

Materials

Paper circles, one per pair · Scissors · Small plates, three per pair · The book's sharing sentence written large

MATH THROUGH THE READING

5. Talking Someone Out of the Bigger Number Integrated · Students take a side on the book's last question, build both slices out of the same size pizza, and argue the case to the reader the book has been addressing until the pieces settle it.

Students take a side on the book's last question, build both slices out of the same size pizza, and argue the case to the reader the book has been addressing until the pieces settle it.

Full lesson - 20 minutes

1. Pick a side and say why (5 min) The teacher reads the book's question. Each student picks three-twelfths or one fourth, stands on that side of the room, and tells a partner from the other side the reason, which must contain a number.
2. Build both slices (5 min) Pairs take two circles of the same size, cut one into four equal parts and one into twelve, and set one fourth beside three twelfths without stacking them yet.
3. Cover one with the other (5 min) Pairs lay the three twelfths onto the one fourth and say what they see at the edges. Anyone who has changed sides moves across the room and says what moved them.
4. Make the case to the book's reader (5 min) Each pair answers the book's question aloud as if speaking to the hungry child the book keeps addressing, giving the answer, the number their reason is about, and what they would tell someone who counted the pieces instead.

Short version - 9 minutes

1. Pick a side and say why (5 min) The teacher reads the book's question. Each student picks three-twelfths or one fourth, stands on that side of the room, and tells a partner from the other side the reason, which must contain a number.
2. Cover one with the other (5 min) Pairs lay the three twelfths onto the one fourth and say what they see at the edges. Anyone who has changed sides moves across the room and says what moved them.

Three Levels

Support

Cut only the twelfths circle and hand the pair a fourth already cut from a matching circle, so the whole task is the covering. The teacher asks the reason as two questions rather than one, first the number and then what it counts. The child gives the position and the number, and the sentence about the other side's reasoning is dropped.

At Grade Take a side with a reason before cutting, cut both circles, run the covering, and give the closing statement including what the other side's number was counting.

Extension

Take the book's second question, one sixth or one eighth, where the two sides are genuinely unequal, and build the argument that would persuade someone who has just learned from the twelfths that these things always come out the same. Say what makes this pair different from the last one.

English Learners Argument syntax is the load here, not fraction vocabulary. The reason clause in English can be led by because, since or so, and the same idea is often carried by word order alone in other languages, so a student may produce a complete mathematical argument that sounds to an English ear like two unconnected statements. Accept the mathematics and rehearse the frame 'I would take ____ because three of the small pieces cover ____ exactly' with the pieces in hand on both blanks. Offer the concession frame separately, 'You counted ____, but the pieces are not the same size', since conceding a point while holding a position is a move that has to be taught in any language. Invite students to make the argument in their strongest language to a partner who shares it before making it in English, so the reasoning is settled before the syntax is attempted.

Materials

Paper circles, two of the same size per pair · Scissors · The book's four closing questions written out, one to a line

At Grade

English Learners

READING AND LANGUAGE

6. The Cat, the Whale, and the Apple Pie Language · Explaining the meaning of a common expression that is not literal Three times this book reaches for an expression that means nothing like what its words say, and once it stops to define the expression before playing with it. Students separate the literal reading from the intended one, name what each expression is actually claiming, and notice that the author supplied a definition exactly where a reader would need one. The work is entirely with meaning, never with the counts on the page.

Full lesson - 20 minutes

1. Read the cat sentence and say the two cats apart (5 min) Read this aloud with a partner. The author uses the word cat twice and means something different each time. Say what the first cat is - the one in the expression - and say what the second cat is. Neither answer is an animal in a bag.

Word preview: expression

When a surprise is discovered too soon, we say the cat is out of the bag. This cat is out of a box. A pizza box. Would it surprise you to discover a cat pizza?

2. Say why the author defined it first (5 min) The author explains the expression in the first sentence before using it as a joke. Say the explaining words aloud, then say what a reader who had never heard the expression would have missed if that first sentence were cut. Write one sentence saying what the first sentence is doing for the reader.

Word preview: discovered

When a surprise is discovered too soon, we say the cat is out of the bag. This cat is out of a box. A pizza box. Would it surprise you to discover a cat pizza?

3. Meet two more expressions that are not literal (5 min) Read these two lines. Neither one is describing a real whale or a real apple pie. For each, say aloud what it actually means, then say which word in it is doing the exaggerating or the comparing.

Word preview: appetite, American If you have a whale of an appetite, you might eat half a pizza.

Pizza has become as American as apple pie.

4. Sort three expressions by what they do (5 min) Put these three lines in front of you and sort them: which one names a size, which one names how common something is, and which one names a secret getting out. Say your sorting to a partner and defend one placement using the words in the line itself.

Word preview: sort

When a surprise is discovered too soon, we say the cat is out of the bag. This cat is out of a box. A pizza box. Would it surprise you to discover a cat pizza? If you have a whale of an appetite, you might eat half a pizza. Pizza has become as American as apple pie.

Short version - 9 minutes

1. Read the cat sentence and say the two cats apart (5 min) Read this aloud with a partner. The word cat is used twice and means something different each time. Say what each one is.

Word preview: expression

When a surprise is discovered too soon, we say the cat is out of the bag. This cat is out of a box. A pizza box. Would it surprise you to discover a cat pizza?

2. Meet two more expressions that are not literal (4 min) Read these two lines. Neither describes a real whale or a real apple pie. Say aloud what each one actually means.

Word preview: appetite, American If you have a whale of an appetite, you might eat half a pizza.

Pizza has become as American as apple pie.

Three Levels

Support

Take the whale line alone. Ask whether a person with a whale of an appetite has eaten a whale, and let the child say no and then say what it does mean. One expression, explained aloud, is the whole task.

At Grade

Explain all three expressions in your own words and name the word in each that carries the exaggeration or the comparison.

Extension

The author defines the cat expression but leaves the whale and apple pie expressions undefined. Say why the cat one needed defining and the other two did not, using what each expression is being asked to do in its own sentence.

English Learners Idioms are the least transferable language in any text and none of these three can be worked out from their parts. Say each meaning plainly before students read, then let them hunt for the words that carry it. Invite a student to name an expression in their strongest language that also means a very large appetite, and to say its literal words - the comparison is often to a different animal, which makes the arbitrariness visible.

Materials

Printed cat page · Notebook · Printed whale line · Printed apple pie line · All three printed expression lines · Three sorting labels: size, how common, a secret out · Word cards: expression, discovered, appetite, American, sort

2. Meet two more expressions that are not literal (4 min)

READING AND LANGUAGE

7. A Pizza Like a Blank Page Language · Testing what a comparison gives a reader that plain description does not The book compares a plain pizza to a sheet of blank paper and then, one clause later, says what the comparison was for: a place to create a picture. Students name both halves, say what the familiar half contributed, hold it against a nearby passage that describes without comparing at all, and finally swap the comparison out to find what breaks. The test is whether a comparison earns its place.

Full lesson - 20 minutes

1. Read the comparison and name both halves (5 min) Read these two lines aloud. The author compares a plain pizza to something from a completely different part of life. Say the two things being compared, and say which one you already know well.

Word preview: plain

A plain pizza is like a blank piece of paper. A great place to create a picture.

2. Say what the comparison hands you (5 min) A good comparison gives a reader an idea they can use. Say what the blank paper tells you about the plain pizza that the word plain by itself did not. Write one sentence beginning The comparison tells me.

Word preview: create

A plain pizza is like a blank piece of paper. A great place to create a picture.

3. Compare it with a description that uses no comparison (5 min) This page describes how dough is made without comparing it to anything. Read it aloud and say what makes it clear anyway. Then say which of the two passages you could picture faster, and what did the work in each.

Word preview: stretches

Pizza chefs need good aim. Some pizza makers flip the dough up into the air. As it spins, the dough stretches out from the center into a thin circle.

4. Replace the comparison and see what breaks (5 min) Take the blank paper comparison and swap in something else - a plate, a window, an empty field. Say the new sentence aloud, then say what your version tells a reader and what it fails to tell them. Decide together which comparison the book should have used.

Word preview: swap

A plain pizza is like a blank piece of paper. A great place to create a picture.

Short version - 9 minutes

1. Read the comparison and name both halves (4 min) Read these two lines aloud. Say the two things being compared and say which one you already know well.

Word preview: plain

A plain pizza is like a blank piece of paper. A great place to create a picture.

2. Say what the comparison hands you (5 min) Say what the blank paper tells you about the plain pizza that the word plain alone did not. Say it as a sentence beginning The comparison tells me.

Word preview: create

A plain pizza is like a blank piece of paper. A great place to create a picture.

Three Levels Support Put a sheet of blank paper on the desk beside the printed line. Ask what a person does with blank paper, then ask what the author says a person does with a plain pizza. One shared idea, said aloud, is enough.

At Grade

Name both halves, say what the blank paper contributed, and say which of the two printed passages you could picture faster.

Extension

The comparison is finished by the fragment that follows it rather than by a full sentence. Say what that fragment does for the comparison, and say whether the comparison would still work if the fragment were cut.

English Learners Comparison with like is structurally simple and transfers widely; what may not transfer is blank paper as a symbol of possibility rather than of emptiness. Put a real sheet on the desk. Rehearse the frame aloud: ____ is like ____ because both _____. A student may swap in a familiar surface from their own experience and explain the swap in their strongest language.

Materials

Printed blank paper line · Notebook · Printed dough page · Word cards: plain, create, stretches, swap

Three Levels

Support